B. Tech - 2

Math-II

Set-1

MATHEMATICS - II

Full Marks : 70

Time : 3 hours

In addition to **Q. No. 1** which is compulsory and answer any **five** from the rest

The figures in the right-hand margin indicate marks

1. Answer the following questions : 2×10

(*a*) Express the following matrix as the 3 cm of a symmetric and a skew-symmetric matrix

$$\begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & -2 \\ -2 & -1 & 1 \end{bmatrix}$$

(*b*) Solve by using Cramer's rule

$$5x - 3y = 37$$
$$-2x + 7y = -38.$$

(*c*) Find the components of the vector $\vec{V}$ with initial point $P(1, 1, 0)$ and terminal point $Q(2, 4, 6)$.

(*d*) Using Vector methods find the area of the triangle whose vertices are at $(1, 1, 1)$, $(4, 4, 4)$ and $(8, -3, 14)$.

(*e*) Find the directional derivative of $f = x^2 + 3y^2 + 4z^2$ at $P(1, 0, 1)$ in the direction of the vector $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$.

(*f*) Calculate div $\vec{V}$ if

$$\vec{V} = xyz(x\hat{i} + y\hat{j} + z\hat{k}).$$

(*g*) Consider the function $f(x) = \pi - x, 0 < x < \pi$. What is the value of the half range sine series at $x = 0$? What is the value of the half range cosine series at $x = 0$?

(*h*) Find the spectrum and eigenvectors of the following matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$$

(*i*) Check whether the following integral is independent of the path of integration. Justify your answer with valid reasons.

$$\int_{(0,0,0)}^{(a,b,c)} (e^z dx + 2y dy + xe^z dz)$$

(*j*) Given that A is a $m \times n$ matrix with rank $= 0$. What can you say about the elements of the matrix A ? Justify your answer with valid reasons.

2. (*a*) Define row equivalence of matrices. Show that row equivalent matrices have the same rank.

(*b*) Find the rank of the following matrix by reducing it to its echelon form

$$\begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

10

3. (*a*) Using Gaussian elimination method test whether the following system of linear equations has solutions. In case solutions exist find them. Justify your answer with valid reasons.

$$5x + 5y - 10z = 0$$
$$2w - 3x - 3y + 6z = 2$$
$$4w + x + y - 2z = 4$$

(*b*) Let $\lambda_1, \lambda_2, ..., \lambda_k$ be distinct eigenvalues of an $n \times n$ matrix A. Show that the corresponding eigenvectors $\vec{x}_1, \vec{x}_2, ..., \vec{x}_k$ are linearly independent. 10

4. (*a*) Calculate the inverse of the following matrix by Gauss-Jordan elimination method, if it exists.

$$\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

(*b*) Find a basis of eigenvectors and diagonalize

$$\begin{bmatrix} -8 & 11 & 3 \\ 4 & -1 & 3 \\ -4 & 10 & 6 \end{bmatrix}.$$ 10

5. (*a*) Show that if f and g are scalar point functions, then

$$\operatorname{div}(f\nabla g) - \operatorname{div}(g\nabla f) = f\nabla^2 g - g\nabla^2 f.$$

(*b*) Show that the following integral is independent of the path of integration. Evaluate this integral

$$\int_{(0,-1,1)}^{(2,4,0)} e^{x-y+z^2}(dx - dy + 2z\,dz).$$ 10

6. (*a*) Using Green's theorem evaluate the line integral

$$\oint_C \vec{F} \cdot d\vec{r},$$

counterclockwise around the boundary C of the region R given by $1+x^4 \le y \le 2$, where

$$\vec{F} = \left[\frac{e^y}{x}, e^y \log x + 2x\right].$$

(b) Evaluate the surface integral

$$\iint_S \vec{F} \cdot \hat{n} dA$$

by using Gauss divergence theorem, where

$$\vec{F} = [\cos y, \sin x, \cos z]$$

and S is the surface of $x^2 + y^2 \le 4$, $|z| \le 2$. 10

7. (a) State Stokes' theorem. Verify Stokes' theorem by finding the surface integral and the line integral for the following data :

$\vec{F} = [z^2, 5x, 0]$, S is the square $0 \le x \le 1$, $0 \le y \le 1$, $z = 1$.

(*b*) The velocity vector $\vec{V}$ of a fluid motion is given by $\vec{V} = x\hat{i} + y\hat{j} - z\hat{k}$. Is the flow irrotational ? Is it an incompressible fluid ? Find the path of the particles. 10

8. (*a*) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}, -\pi < x < \pi.$$

Hence show that $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(*b*) Obtain the half range cosine and sine series for $f(x) = x$ in the interval $0 \le x \le \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 10

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